Fluid Mechanics for Civil and Mechanical Engineering
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Lecture No 12
Bernoulli Equation and its Applications

Welcome all of you for this very interesting lectures on Bernoulli Equation and its Applications. The last class we discuss about Bernoulli equation starting from its history of Bernoulli equation. Very briefly, I can say that because of the Bernoulli equation, having a simplification to the fluid flow problems, the industrial revolutions what it happened in Europe, the contribution of Bernoulli equation also helped a lot to design pipe flow, channel flow in Europe after this equations was suggested long back in 1752.

Now, today I will give a very simple way representation of this Bernoulli equation, how we can use for real fluid flow problems with some correction factors or we can use this Bernoulli equation as hydraulic gradient line, energy gradient line and we can apply these equations for a systems having pump and the turbine. So, basically today, I will talk more applications and how we can use the Bernoulli equation for real fluid flow problems. That is the basic concept what I will do and these are the reference books.

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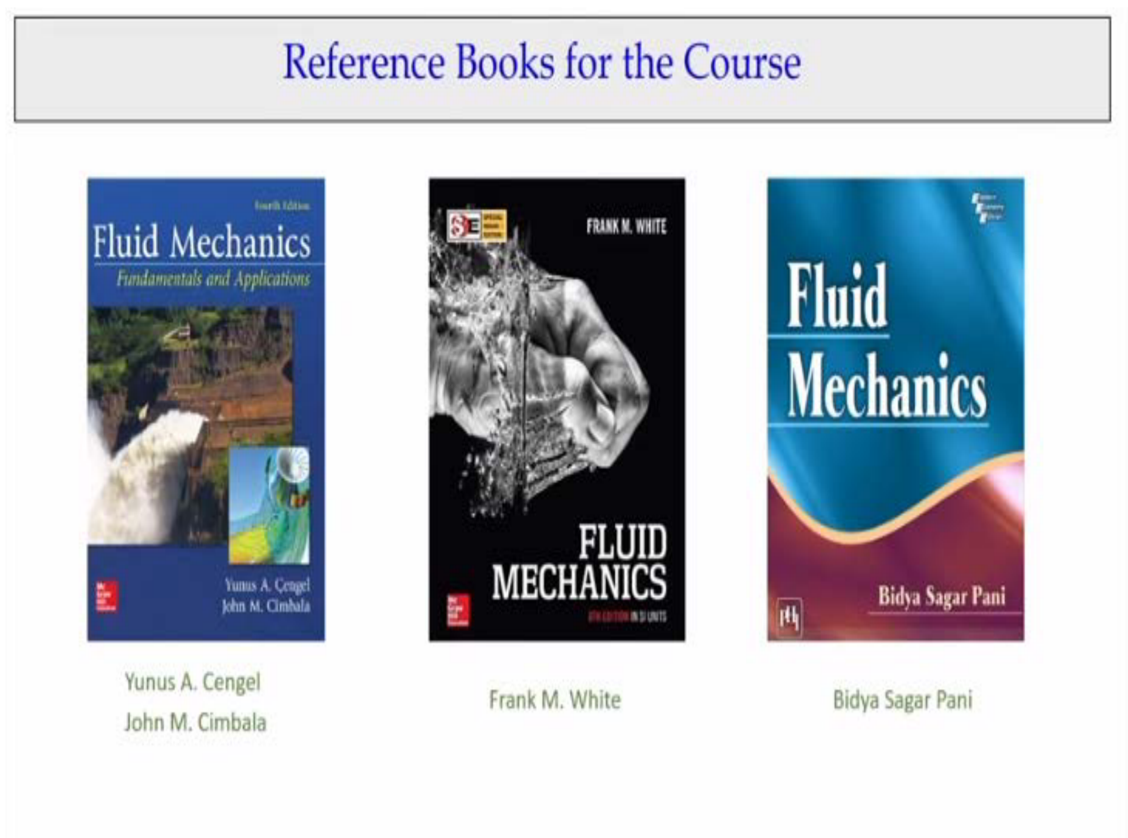

And most of these materials partly we follow Cengel, Cimbala and F.M. White book.

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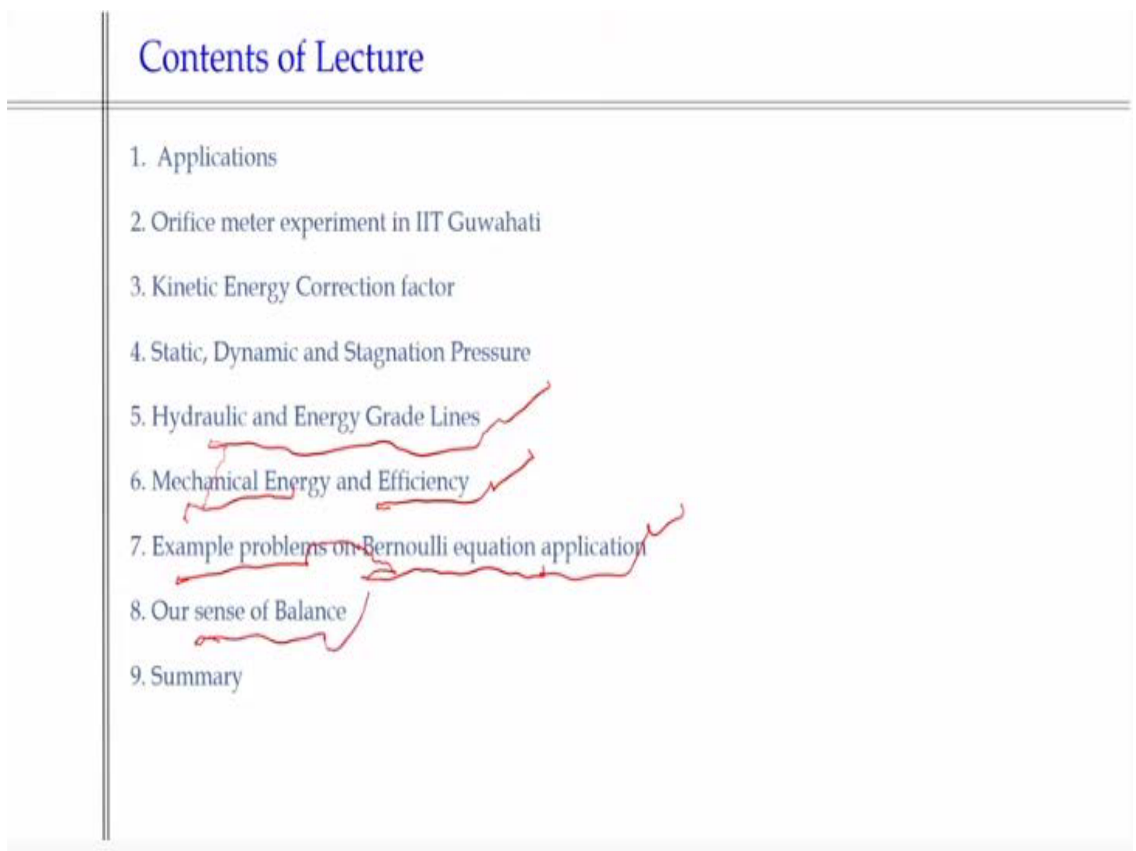

I will start with the applications, very interesting applications I will show to you, then I will go for orifice meter experiment in IIT, Guwahati, then I will talk about kinetic energy corrections factors. That means, for a non-uniform distribution of flow, when you apply this Bernoulli equations, we need to have a corrections factors if we are using average velocity. That correction factors for Bernoulli equations.

Fourth part, I will talk about how we can define the three different types of pressures; static, dynamic and stagnation pressures. Then will come hydraulic and energy gradient lines, that is the basic concept what we will talk about. Then I will talk about if we have a pipe flow systems, with a series of pump turbine systems, then how we apply it and how we can quantify different energy mechanical energy also the efficiency to the fluid flow problems.

Then, we will solve around four fluid flow problems, which are the gate and the engineering service problems will solve, which is part of the Bernoulli equations applications. Then concluding this lecture, we talk about, how our sense of balance is there. So okay, how it is works.

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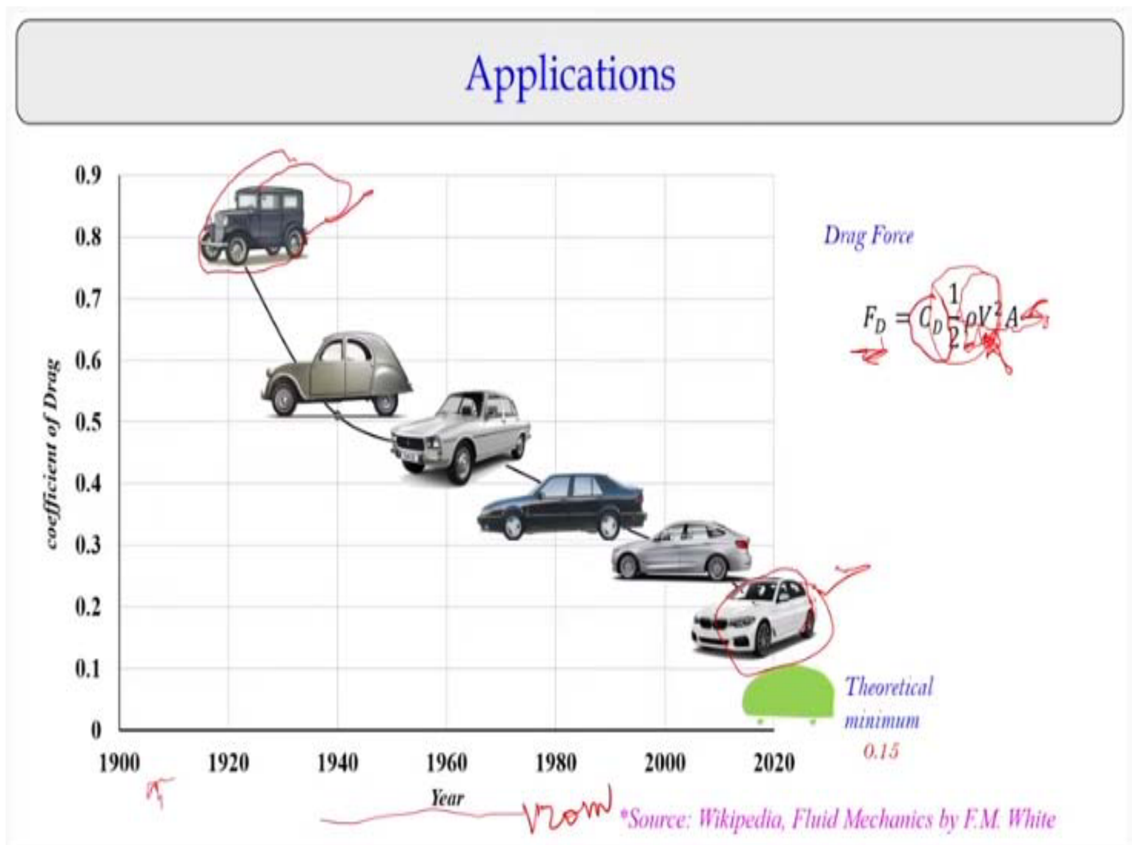

So, let us go back to very interesting examples, as you know it people say as of now, we have very fuel efficient car. That means the mostly car we talk about the drag force. The more the drag force, then you have to have more fuel to be spent it, fuel to burn it. So the basic idea is to, because V stands for the design velocities, area is a projected area, which is more or less constant. So, rho is the density of air. Only the C_D can be changed. That is what it happened the evolution of the C_D change from almost 120 years, okay.

Drag Force

$F_D = C_D \frac{1}{2} \rho V^2 A$

Starting from 1800s to 2020. So, the coefficient of drags, which earlier is close to the 0.8, that means the force is equal to 0.8 times of rho V square into the area that becomes reduced to the 0.15, theoretical minimum value, very close to theoretical value. And that because of you can see the shape of the cars, okay. And the shape of the cars are changing, the streamline, the drag force patterns are changing the flow around these cars when you are moving with a design velocity.

That is what it changes it, with a design velocity. The cars is moving with the V, which is the design velocity. On that, what is the drag force component. So that is what if you look it, we have achieved, within 100 years, the theoretical minimum value which is 0.15, very close to that. And you look the shape of the cars what today we have, which if you move with a design speed, then we will have a drag coefficient close to 0.15 and which is a strength of the fluid mechanics evolved the automobile sectors with a very efficient the fluid cars.

What is today is available and their drag ratio starts from the coefficient of drag C_D from point 0.8 to 0.15. That means as equivalent to 1/6. So, for example the 6 liters of the car if it is taking this one for one particular distance to travel it, now, we need just 1 liter, the same distance can be traveled because of the drag coefficient is almost 1/6. So, this is what is the evolutions happened is from the C_D equal to 0.8 to 0.15.

And that because of series of the experiments done for the automobile sectors to find out what could be the best shape of the cars. So that, the coefficient of drag should be minimum or should be coming to closer to theoretical minimum value of 0.15. So that is what it happened in the evolutions of what has happened. And that is the reasons we have a fuel efficient cars available nowadays and with a C_D value close to the theoretical minimum and this is what is possible because of the fluid mechanics as evolution with last 120 years.

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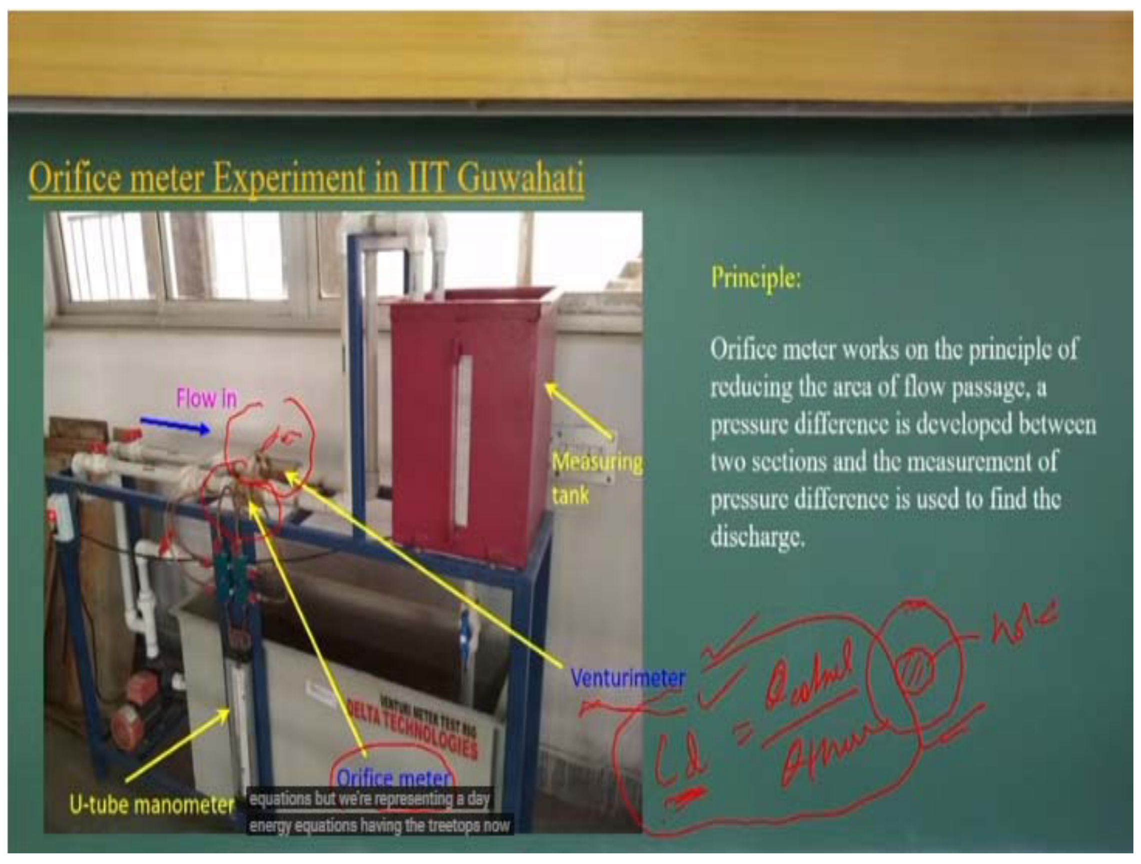

Now come back to the where simple experimental setups, we generally do the measuring the flow in a pipe, either in a venturimeter or the orifice meter. The orifice meter is a smalle device, that means it will be a circle and there is a hole inside this. This is called the orifice. In case of the venturimeter you have a diverging zone and converging zone. But in case of orifice, we do not have a diverging and converging zone, only you have a reductions of the flow area that is what is the orifice meter.

That means you have a simple plate with a hole with a constant, a particular diameters and that is what is fixed here. So, we can measure the pressure difference at two locations; one is at the fiber locations where the cross sections is really is less and another is the off stream locations. That pressure difference and if you apply simple Bernoulli equations and the mass conservation equation, you can find out what will be the discharge passing through venture or orifice meters.

But as I said it earlier, that whenever flow goes through these venturimeter or orifice meter, there will be energy losses. Because of the energy losses, they will be difference between theoretical discharge and actual discharge. Since the Bernoulli equations what we applied, we do not consider the energy loss components. We consider there is no energy losses, okay. No the frictional resistance is more or there is no vortex formations and all. But in the real fluid flow, there is energy losses.

Because of that, what do you will have, your theoretical discharge compute from the Bernoulli equations and the mass conservations will be more than actual discharge. So actual discharge, the difference between these, we introduce a coefficient of discharge, that means the CD.

$$c_d = \frac{Q_{actual}}{Q_{theoritical}}$$

Coefficient of discharge is C_D here, it is not a coefficient of drag, please do not have a conclusion between the two terms.

If you look at the device where we have a setup of venturimeters and orifice meters, the basically in a venturimeter we have a converging zones and diverging zones and in orifice we do not have a converging diverging zones, but we just incorporated a plate with holes, okay, with a internal hole of this. Because of that, it also create, the converging zones and the diverging zone.

And because of that, we can have a energy losses in a flow systems when the streamline are converging and the diverging, definitely there will be energy losses with this systems. As the energy losses will be there, the theoretical discharge computed using Bernoulli equations and the conservation equations will be higher than the actual discharge. That is the reason, we introduce the coefficient of the discharge, the C_D value, which is the ratio between actual discharge by theoretical discharge.

So, we introduced the coefficient of discharge in this and many of the times we have to compute it, what is the coefficient of discharge. So now if you can try to understand it that many of the times we do not go inside in this fluid flow problems within the venturimeter or the orifice meter that how the flow streamlines, how the energy dissipater happens, that what we do not look it, that in depth. But in terms of these flow patterns, what is it the gross characteristics in terms of energy losses, in terms of reductions in a discharge.

That is what we introduce as a C_D coefficient, the coefficient of discharge. So, these are gross representations of change of the flow patterns, the pressure and velocity patterns. Because of that and flow structures. We need we do not look it very microscopically, how it is changing it, but as a gross representation, we just look it the coefficient of discharge as the relationship between the theoretical and the actual. There is a link between now experimental work and analytical farm work.

So, the Bernoulli equations is strength there that this is the equations we can easily incorporate the experimental relationship into these Bernoulli equations. So, that is the strength of the Bernoulli equations as compared to the other Euler forms or other Navier-Stokes equations form, where Bernoulli equations are very suitable, appropriate to incorporate any energy losses, energy to this a fluid flow systems or taking out from the fluid flow system.

All we can incorporate it in a Bernoulli equations looking as energy conservation equations, looking as a energy conservation equations. But we remember, this is a equations for along a streamlines, is a momentum equations, but we are representing as a energy equations, having the three talks.

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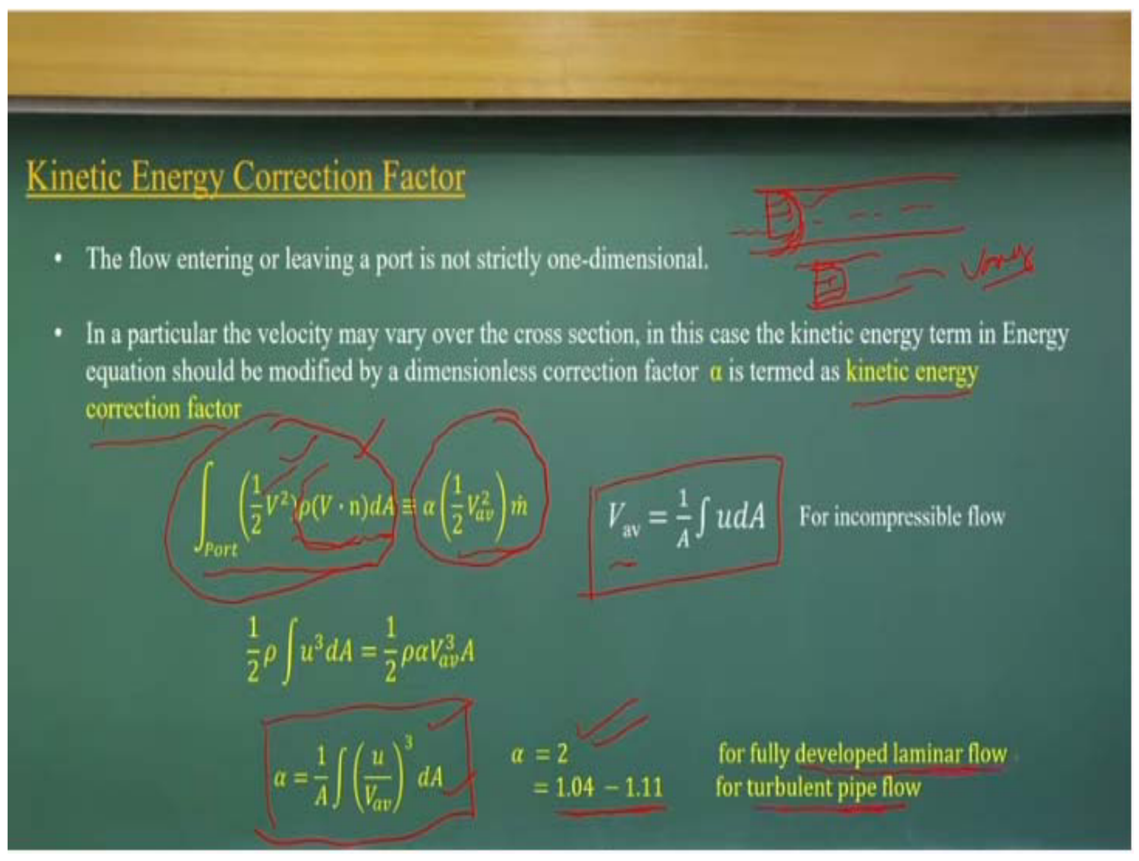

Now, let us come back that whenever you have a fluid flow problems, like fluid flow through a pipe. So definitely the velocity distributions is not uniform. The velocity distributions will be change it, from laminar to the turbulent. The turbulent will have logarithmic profiles and the laminar will be parabolic profiles. So velocity distribution changes, so what we do it that, it is very difficult for us to put it as the average velocity to compute the kinetic energy.

Because the flow within these pipes follow, having a parabolic flow velocity distribution. Now, we are looking it, if I consider the flow velocity distributions. Because of that velocity distribution, what is the total amount of kinetic energy? I can compute it with simple integrations, considering the velocity distributions. Then I will compute it, if I consider the average velocity, what will be the total kinetic energy. And since I am going to use average velocity, not the velocity distributions.

$$\int_{Port} \left(\frac{1}{2}V^2\right)\rho(V \cdot \mathrm{n})dA \equiv \alpha\left(\frac{1}{2}V_{av}^3\right)\dot{m}$$

$V_{\mathrm{av}} = \frac{1}{A}\int u dA$ For incompressible flow

I apply a correction factor for that, which is called kinetic energy correction factors. Let me repeat these things, what we are looking at that in a real fluid flow problem, always we have flow is not uniform, non-uniform distribution. that means it will have a either a parabolic distribution, logarithmic distributions. It depends upon the flow, through pipes, or open channels. The flow velocity distributions varies in a space.

So, when you apply the kinetic energy, we look at the total kinetic energy, which is we can integrate over that a small dA element with a. This is a mass flux, rho V dot n is a mass flux and this is a half rho V square is the mass flux into V square by 2 is a kinetic energy flux. That is what we will do the integrations over the total area dA and we have the kinetic energy. So, if you look it, the kinetic energy computations, using the V square, V average values will have the kinetic energy computed using average velocities.

Times of alpha, alpha is kinetic energy correction factors. The basically what it indicates that, if you look it, compute the kinetic energy best on average velocity, then what could be the correction factors for non-uniform distributions of velocity in the particular flow field condition. So, if I equating these things, then my alpha will comes like this part, it is very simple part.

$$\frac{1}{2}\rho\int u^3 dA = \frac{1}{2}\rho\alpha V_{av}^3 A$$

As you know, V average we compute, is average velocity, we can compute like this.

$$\alpha = \frac{1}{A}\int\left(\frac{u}{V_{av}}\right)^3 dA$$

$$\alpha = 2 \qquad \text{for fully developed laminar flow}$$

$$= 1.04 - 1.11 \qquad \text{for turbulent pipe flow}$$

So alpha can consider it V average u to the power 3 or cubic power dA and this integrations will give us the alpha. For fully developed laminar flow, this alpha value is about 2. Turbulent pipe flow, this is what varies from 1.04 to 1.1. That is what is the biggest problems many of the fluid mechanics book, they do not give the alpha value, they consider the alpha value is close to the 1 or indirectly they represent that flow is turbulent. The alpha value variations of 1.04 to 1.11, they do not consider it.

But that is not correct, any fluid flow problems, we should compute what is the alpha value and whenever we apply the Bernoulli equations, considering the velocity distributions, the kinetic energy, correction factors needs to be done it and that is the value which is varies for the turbulent pipe flow and for the laminar flow, which is equal to 2 value.

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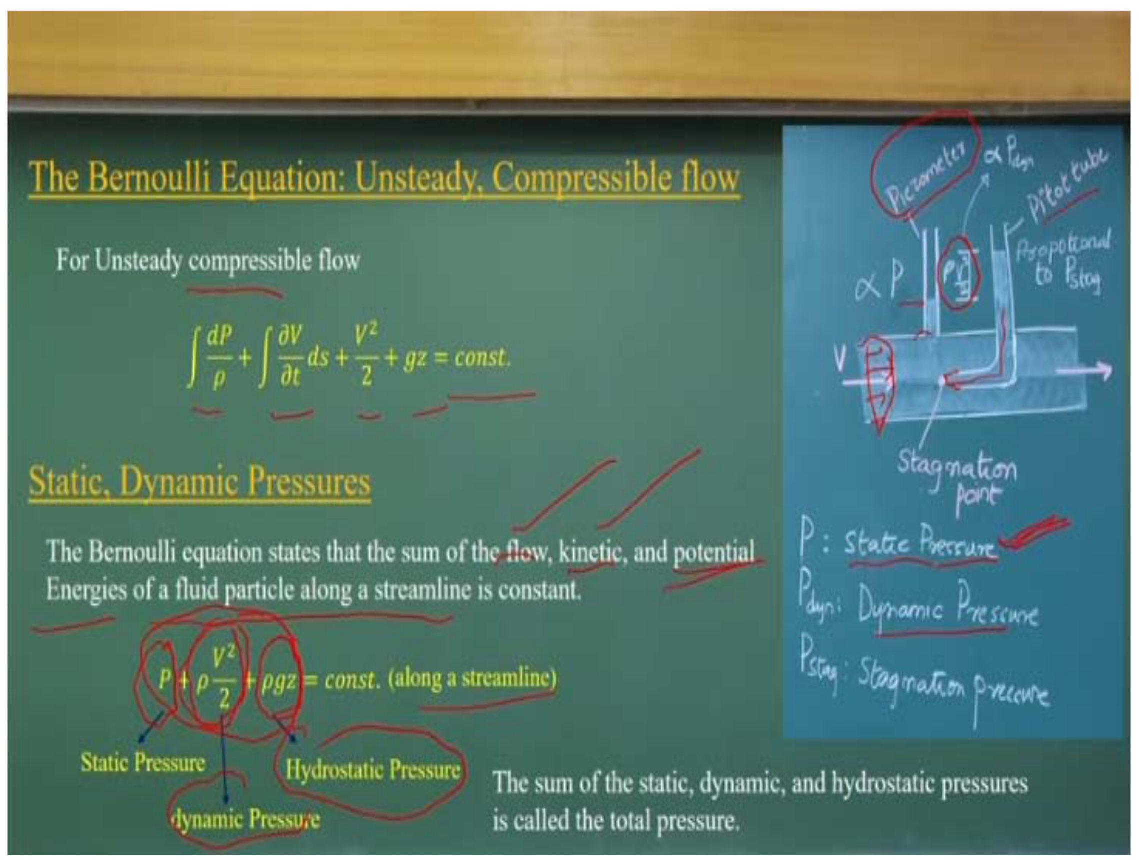

Now, let us come back to this Bernoulli equation what we have derived, that in case of unsteady compressible, will have the two integrals component.

$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = const.$$

But in case we make it the steady and incompressible, then we get it these three forms as earlier I discuss it, this is summations of three energies, flow energy, which is because of the pressure at that locations.

How much work is done by that pressure, that is what will be the flow energy. You have a velocity field, which have a kinetic energy and the fluid particles or virtual fluid balls staying a particular height and from a **(()) (19:30)**, which gives potential energy. So, three energy along a streamline is a constant. The sum of these three energy flow, kinetics and the potential energy of a fluid particles along a streamline is constant. That is what we have written it.

$$P + \rho \frac{V^2}{2} + \rho g z = const.$$

Now, in terms of pressures, if you talk about, one is static pressure, that is the presses act on this fluid particles another is the dynamic pressure because of the velocity components. Please remember this $\rho \frac{V^2}{2}$we have used when you come the drag force, okay. And we have $\rho g z$, which is the hydrostatic pressure. So, we have three pressure components, in terms of pressure if you look it. In terms of energy, we can define it is a flow energy, kinetic energy and potential energy of a fluid particles along a streamline is a constant.

Now, let me demonstrate it if I have pipe flow, flow is going through this, it has a velocity distribution. As you know it, the velocity near to wall is 0 and whatever the height of the fluid go, this is what will be represent is me the pressure. That is what will be represent me the pressure head, that is what the static pressures. So, basically this height will be the static pressure and that is what we measure through piezometers. But, if I insert a tube and put it inside this.

And as the flow is moving with velocity V and stuck over this, since it is a constant, it is a fixed point, the velocity has to be zero at that point. So, the stagnation point will be work on this. So, because of that, what will be happen it, the two energy components what we have the flow energy and the kinetic energy that what convert to as a head, as equivalent water head or the liquid head will get it that. So difference between these two will give us $\rho \frac{V^2}{2}$.

That is what is the kinetic energy head. The difference between this Pitot tube and the piezometer will give us the difference of these energy $\rho\frac{V^2}{2}$. But you can say that, what about the potential energy. These two energies are so high that, we do not consider the z difference what we have that may not have a that significance. That is the reason, we do not consider the potential energy part, but we can measure. When you have a pipe flow we can measure, using the piezometer you can measure the pressure.

Using the pitot tube, which **(()) (22:46)** can insert and have a sharp point to rise the fluid inside the tube. That is what measures both the components, the dynamic pressure as well as the static pressures. The dynamic pressure and the static pressure. Since we know the static pressure, you just compute what is will be the dynamic pressure. The stagnation pressure is a sum of static pressure and dynamic pressure.

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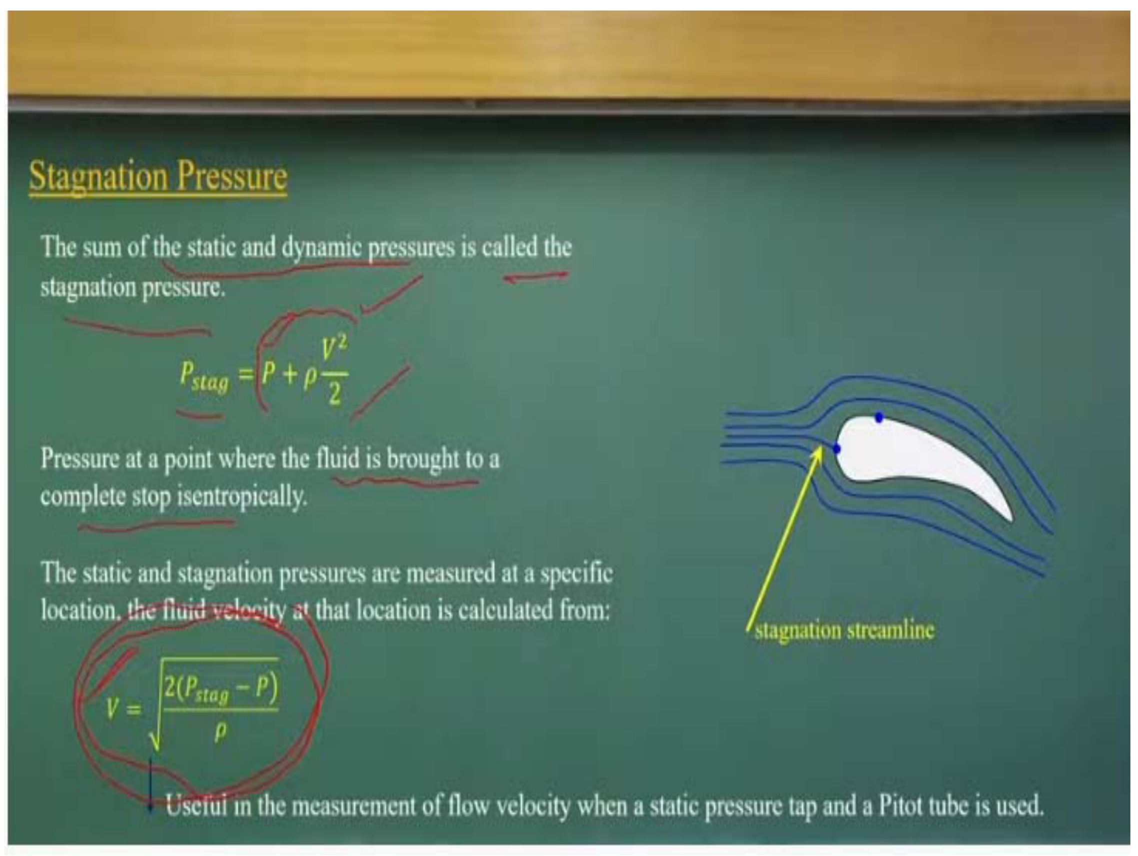
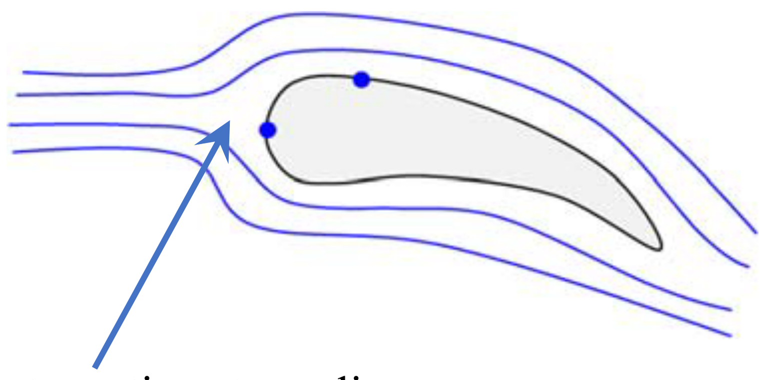


Now, if you look it that same concept what we are talking about. As I say that the sum of the static and dynamic pressure is called the stagnation pressure. That whenever you have an object, which is at the stationary conditions, it is at rest conditions. At that point, will have the, the pressure will be the, because the velocity at that point of the fluid flow have to the zero. So we get is the stagnation pressure, we will have a two component.

$$P_{stag} = P + \rho\frac{V^2}{2}$$



stagnation streamline

The static pressure component and the dynamic pressure component. So that is what we get it, because pressure at point where fluid is brought to a completed stop, then we will have a stagnation pressure will be two components, one is a static pressure component another is dynamic pressure component and those equations we can rearrange it to compute it what will be the velocity. I think this is what is used in even in modern era, in any aircraft wings, you can see there are pressure sensors and based on these pressure sensors, we can easily compute it, what will be the air speed.

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

So to measure the air speed in any modern aircraft, still also use the same concept of the measuring the pressure and as you measure the pressures at this stagnation point and you know the atmospheric pressures, then we can easily find out what could be the airspeed is coming that. This is the equations used for this to measure the flow velocity when static pressure staff at pitot tube is used.

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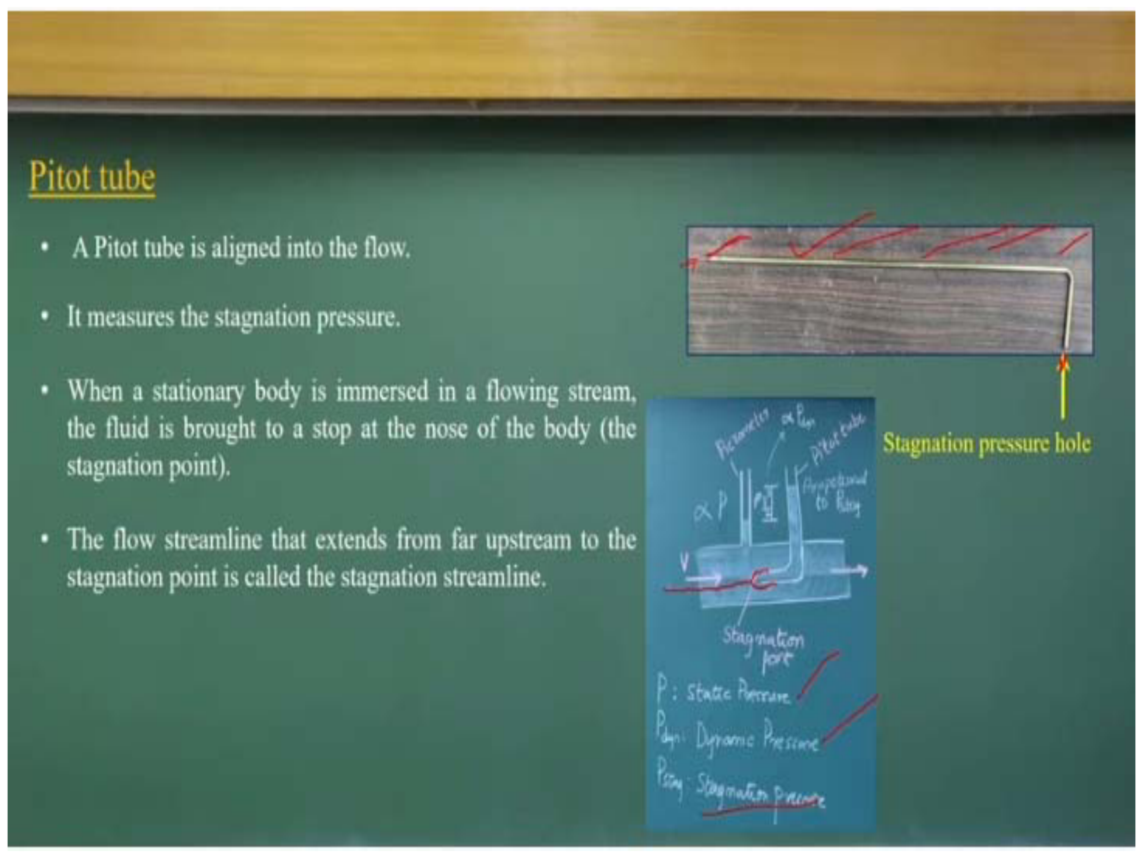

This is the same concept of the pitot tubes as I explained to you, the basically it measures the stagnation pressures, which have both the components and if you have the piezometer, then you can differentiate and find the dynamic pressures and once you know the dynamic pressure, you can compute what could be the velocities. The flow streamline that extend from upstream to the stagnation point is called stagnation streamline, the basically this streamline, which is just come it hit over the mouth.

Just pinpoint of this hole, this is what the pitot tube, this is not a big instrument, this is a small tube inclined here. So, there is a pressure hole, at this point. This is the pressure holes to measure it, this is the points where the stagnation pressure is develop it and that is what will be the two components, static pressure and dynamic pressure and using these with the help of the manometer we can measure the pressure.

And once you know the pressures, you can compute it dynamic pressures and you can compute the velocity. It is a just a applications of Bernoilli's equation.

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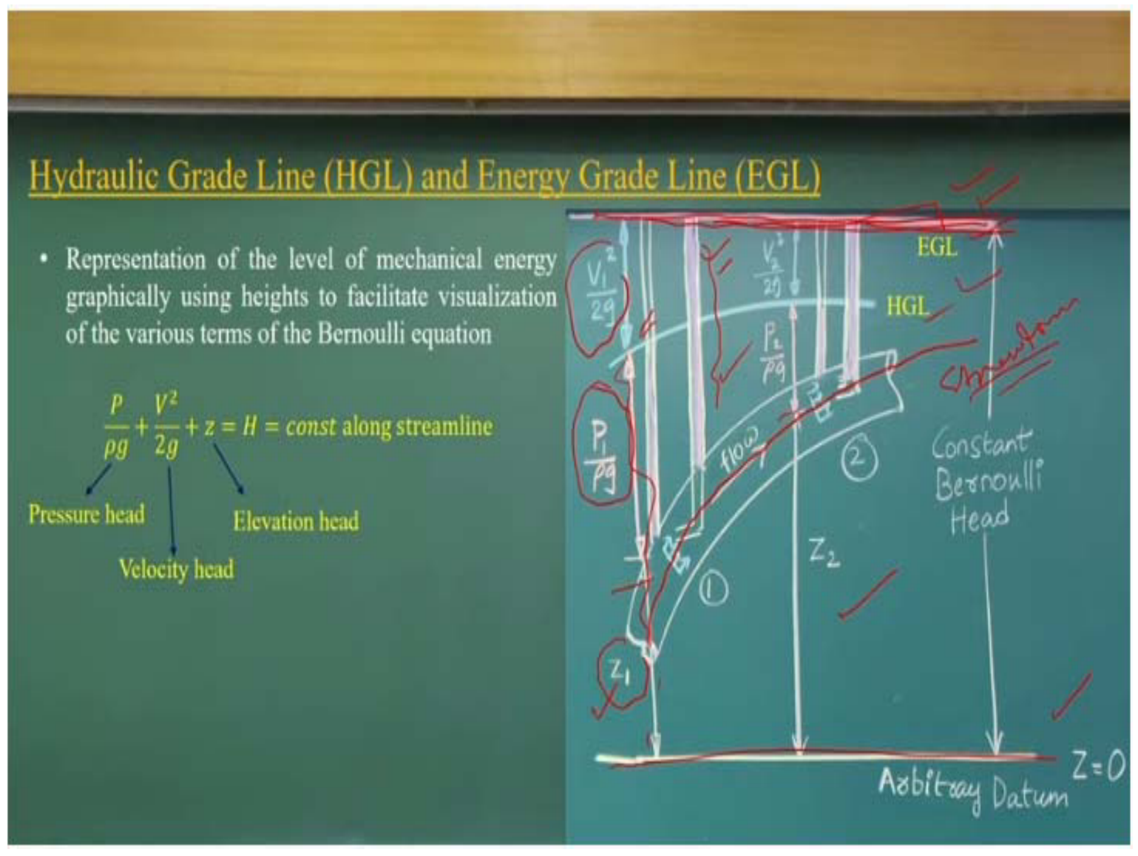

Now, another interesting applications of Bernoulli equations is that energy gradient line and the hydraulic gradient line. This is very great simplifications of the fluid flow problems. Like you may have fluid flow problems with pipe arrangement, the dock arrangement and all. Any flow, as I say that, it can have the flow distributions. But as total energy, along a streamline, if you can draw a streamlines and you want to quantify how these energies are changing it, okay,

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = const \text{ along streamline}$$

That itself as you know it, fluid flows from higher energy to the lower energy, not the higher elevation to lower elevation. Please do not have that things. The fluid flows from higher energy to the lower energy. As it flows from any pipe flow, the channel flow, any doc flow, okay, always there will be a flow which starts from the higher energy to lower energy. Whenever flow from higher energy to lower energy, definitely there is an energy loss is happens.

And these energy losses if you can quantify experimentally, that is what I will discuss when I discuss the pipe flow, how very interestingly this energy losses in the pipe are quantified and then you apply the Bernoulli's equation for designing total pipe network. Same way if you know it in a different type of open channel flow, how much of energy losses is happening, you apply this Bernoulli's equation. Also, you can design the open channel flow.

The basic concept here is called that we should always gauge energy gradient line, hydraulic gradient line. So, any flow patterns, we have to draw the energy gradient line and hydraulic gradient line. What is energy gradient line? Now, we are representing the Bernoulli's equation again in a different forms, it is the same things, okay. Same energy we talk about in terms of head, in terms of meter, okay. That means as equivalent if I have any liquid will be there, how much the lift will be there because of the pressure.

Because of the flow energy, because of the kinetic energy, because of the potential energy. That is the reason, we define it as a head. The pressure head, the velocity head, and the elevation head. It is nothing else, they are all three energy, the flow energy, kinetic energy, and the potential energy, constant along a streamline. And this energy gradient line and the hydraulic lines, what the difference between that. Energy gradient lines consider all these points.

That means, you have Z_1, this is a flow energy, this is the kinetic energy. If you do not have the kinetic energy or the velocity head, then we can have the piezometer lines. If you consider the velocity head, then we have energy gradient. So, always we should have a flow from higher energy to lower energy, okay. This gauge is not that appropriate, that should be the gradient of energy gradient lines. Otherwise, it is not possible to have a flow, okay.

If having a flow, if there is flat energy gradient line, there will be no flow. There is no flow when you have a constant energy gradient, there is no slope of energy gradient line. So, definitely there will be scope of energy gradient line, okay. And that what is drive the flow. So, you have energy gradient line and you can have a pitot tube and the piezometer to measure the hydraulic gradient and the energy gradient, and the energy gradient line and then we can sketch it as energy gradient line to get the hydraulic gradient line.

And many of the times, we can consider a **(()) (30:50)** from where you can compute the Z_1 Z_2, the elevations head of the, that means we define a streamline, then we are just drawing the energy gradient lines for this streamline, for this streamline we are drawing the energy gradient line, we are drawing the hydraulic gradient lines. The hydraulic gradient lines is not, it does not consider velocity head, it just consider pressure head and the elevations head.

Just like you just put a piezometer, then you get it what is the pressure head, also the elevation head is know to us that the what will come it. As soon as you put the piezometers whatever the point to will come it, that what will define us hydraulic gradient line, if I put a pitot tube, the line what will by the top surface will be the energy gradient line. So, whenever we design the pipe flow or channel flow and all, we need to draw energy gradient line and the hydraulic gradient line.

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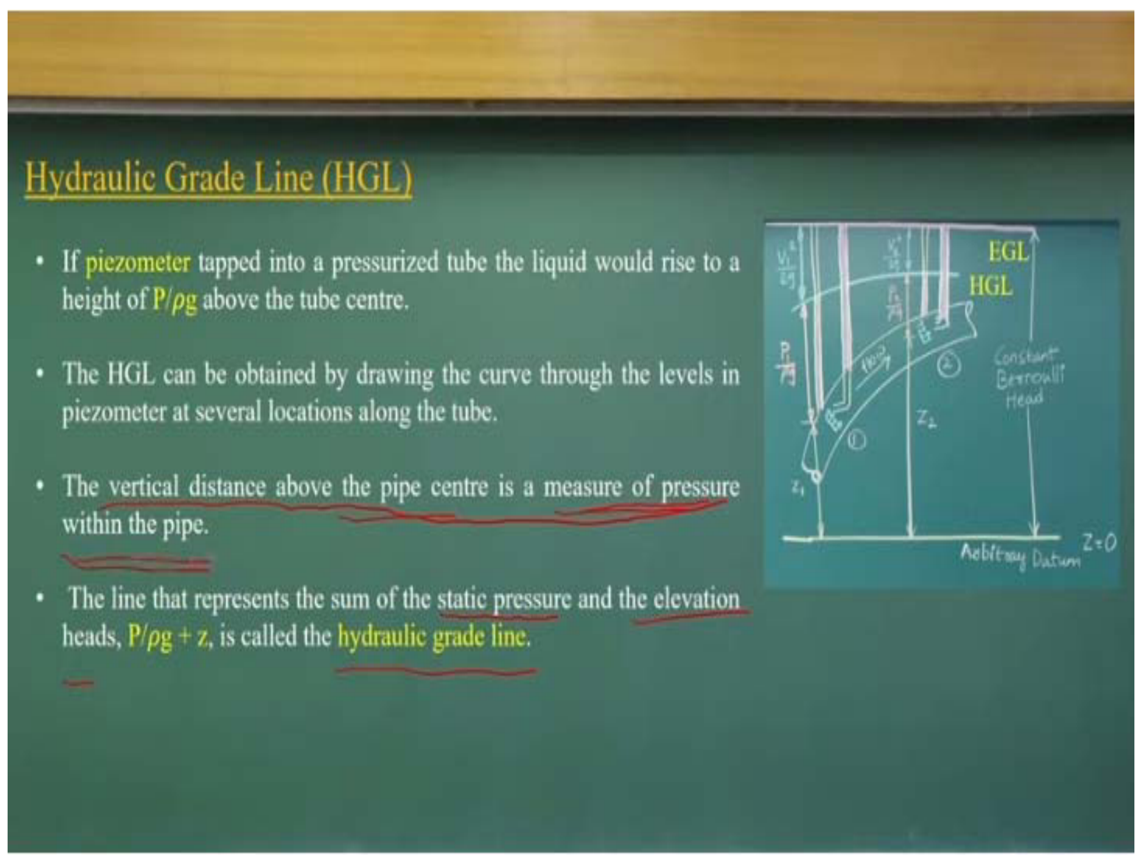

That is what I have explaining it that, so the basically piezometers and the hydraulic gradient line what I have said it and the hydraulic gradient line has some of the static pressures and the elevations. And the vertical distance above the pipe center is measure of the pressure within the pipe, okay. Within the pipe, what is the distance above the pipe centers, is a measure of the pipe pressure within this.

$P/\rho g + z$, is called the hydraulic grade line

What is talking about this sentence is that even if you are putting it here, what is the pressure we are getting it that what is the pressures within the pipe, it is measure it, more or less the same value, the elevations will not be that difference.

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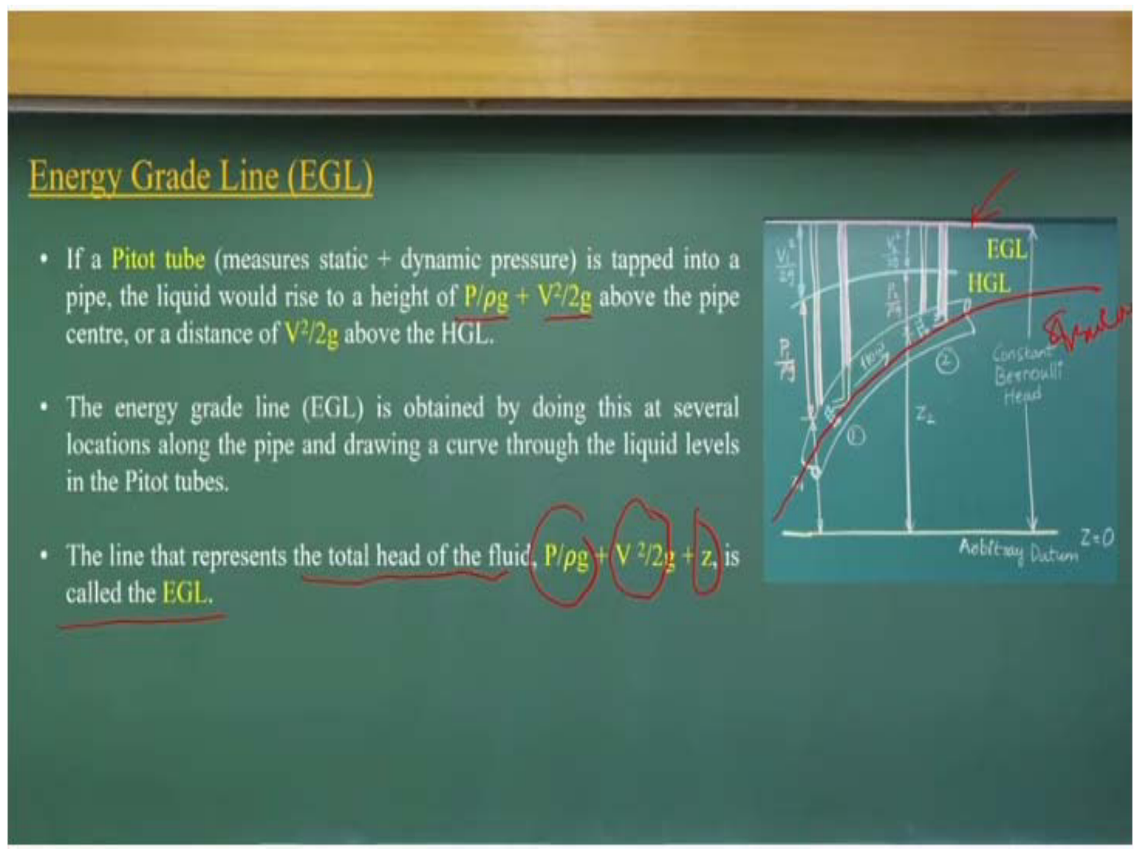

This is what I explain it, if a pitot tube is tapped in the pipe, the liquid could rise in height, which will have the static head and the dynamic head above the pipe and that will be above of dynamic pressure of the $V^2/2g$. The velocity head part and that is what we can draw it, represents a total head using energy gradient line, which represents the total head of the fluid flow energy, kinetic energy and the potential energy, whenever we draw a streamline, we can compute the energy gradient line, hydraulic gradient line.

So, again I continue to say that energy gradient line should have some slope, so there is will be the flow. The flow will be from higher energy level to the lower energy level. The hydraulic gradient may have the difference, okay that is not a big problems, but the flow goes from high energy level to low energy level. The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the EGL

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